CAPÍTULO

1

Los números reales

[1]

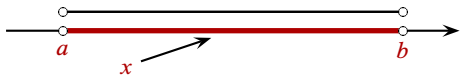
1.5 Intervalos

1.5.1 Tipos de intervalos

Supongamos que tenemos dos números reales a & b, tales que $a < b$.
Se definen cuatro tipos de intervalos:

1. Abierto

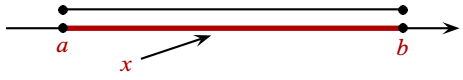
   - $(a,b) = \{ x \in \mathbb{R} \mid a < x < b \} = \{ x \in \mathbb{R} \mid x > a \quad \& \quad x < b \}$.



   En esta representación del intervalo (a,b) las circunferencias expresan que "x" no toma ni el valor de "a" ni el valor de "b".

2. Cerrado

   - $[a,b] = \{ x \in \mathbb{R} \mid a \leq x \leq b \} = \{ x \in \mathbb{R} \mid x \geq a \quad \& \quad x \leq b \}$.



[1] canek.azc.uam.mx: 22/ 5/ 2008

El círculo en a indica que "x" puede tomar el valor de "a". Lo mismo ocurre para "b".

3. Semiabierto o semicerrado

- $[a,b) = \{\, x \in \mathbb{R} \mid a \leq x < b \,\} = \{\, x \in \mathbb{R} \mid x \geq a \quad \& \quad x < b \,\}$.

- $(a,b] = \{\, x \in \mathbb{R} \mid a < x \leq b \,\} = \{\, x \in \mathbb{R} \mid x > a \quad \& \quad x \leq b \,\}$.

El centro de un intervalo es su punto medio: $\dfrac{a+b}{2}$.

4. Infinitos:

- $(a,+\infty) = \{\, x \in \mathbb{R} \mid x > a \,\}$.

- $[a,+\infty) = \{\, x \in \mathbb{R} \mid x \geq a \,\}$.

- $(-\infty,a) = \{\, x \in \mathbb{R} \mid x < a \,\}$.

- $(-\infty,a] = \{\, x \in \mathbb{R} \mid x \leq a \,\}$.

Ejemplo 1.5.1 *Algunos intervalos son:*

▼

1. $(-2,3) = \{\, x \in \mathbb{R} \mid -2 < x < 3 \,\}$.

2. $[-3,2] = \{\, x \in \mathbb{R} \mid -3 \leq x \leq 2 \,\}$.

3. $(1,5] = \{\, x \in \mathbb{R} \mid x > 1 \quad \& \quad x \leq 5 \,\}$.

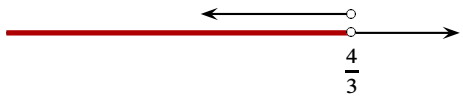
4. $\left[0, \frac{3}{2}\right) = \left\{ x \in \mathbb{R} \mid x \geq 0 \quad \& \quad x < \frac{3}{2} \right\}$.

□

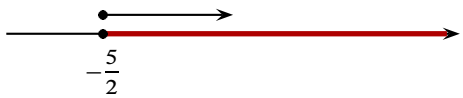
Ejemplo 1.5.2 *Algunos intervalos infinitos son:*

▼

1. $\left(-\infty, \frac{4}{3}\right) = \left\{ x \in \mathbb{R} \mid x < \frac{4}{3} \right\}$.



2. $\left[-\frac{5}{2}, +\infty\right) = \left\{ x \in \mathbb{R} \mid x \geq -\frac{5}{2} \right\}$.



3. $(-\infty, 0] = \{ x \in \mathbb{R} \mid x \leq 0 \}$.

4. $(0.1, +\infty) = \{ x \in \mathbb{R} \mid x > 0.1 \}$.

□

1.5.2 Operaciones con intervalos

Debido a que los intervalos son conjuntos (de números) podemos realizar con ellos las operaciones que se efectúan con cualquier par de conjuntos. Mencionaremos tres: unión $\cup$, intersección $\cap$ y diferencia $-$.

Si I_1 e I_2 son dos intervalos cualesquiera, entonces:

1. Unión de I_1 e I_2
$$I_1 \cup I_2 = \{ x \in \mathbb{R} \mid x \in I_1 \text{ o bien } x \in I_2 \}.$$
2. Intersección de I_1 e I_2
$$I_1 \cap I_2 = \{ x \in \mathbb{R} \mid x \in I_1 \ \& \ x \in I_2 \}.$$
3. Diferencia de I_1 e I_2
$$I_1 - I_2 = \{ x \in \mathbb{R} \mid x \in I_1 \ \& \ x \notin I_2 \}.$$

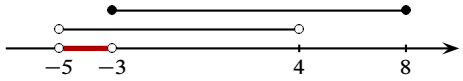
Ejemplo 1.5.3 *Si se toman los intervalos* $I_1 = (-5, 4)$, $I_2 = [-3, 8]$, $I_3 = (-\infty, 2)$, $I_4 = [-1, +\infty)$ *e* $I_5 = (4, 7]$, *entonces:*

▼

1. $I_1 \cup I_2 = (-5, 4) \cup [-3, 8] = (-5, 8]$.

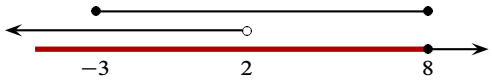
2. $I_1 \cap I_2 = (-5, 4) \cap [-3, 8] = [-3, 4)$.

3. $I_1 - I_2 = (-5, 4) - [-3, 8] = (-5, -3)$.

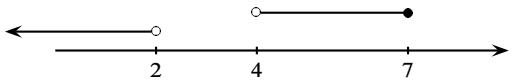


4. $I_3 \cap I_4 = (-\infty, 2) \cap [-1, +\infty) = [-1, 2)$.

5. $I_2 \cup I_3 = [-3, 8] \cup (-\infty, 2) = (-\infty, 8]$.

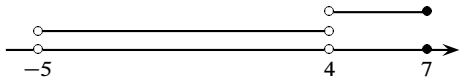


6. $I_3 \cap I_5 = (-\infty, 2) \cap (4, 7] = \varnothing =$ el conjunto vacío.



7. $I_1 \cup I_5 = (-5, 4) \cup (4, 7] = (-5, 7] - \{4\}$.

8. $I_1 \cap I_5 = (-5, 4) \cap (4, 7] = \varnothing$.



9. $I_5 \cup I_2 = (4, 7] \cup [-3, 8] = [-3, 8]$.

10. $I_3 \cup I_4 = (-\infty, 2) \cup [-1, +\infty) = (-\infty, +\infty) = \mathbb{R}$.

□

Comentario. En las operaciones con intervalos se debe tener presente lo siguiente:
Si I es un intervalo cualquiera, entonces:

1. $I \cup \varnothing = I$.
2. $I \cap \varnothing = \varnothing$.
3. $I \cup \mathbb{R} = \mathbb{R}$.
4. $I \cap \mathbb{R} = I$.

Si A y B son intervalos (conjuntos) cualesquiera, entonces:

1. $A \cup B = B$ si $A \subset B$ (es decir, si A es un subconjunto de B).
2. $A \cap B = A$ si $A \subset B$.

Ejercicios 1.5.1 *Soluciones en la página 8*

Escribir las siguientes desigualdades con notación de intervalo y representarlas geométricamente:

1. $-4 \leq x < 3$.
2. $x > -12$.
3. $x < 0$.
4. $\pi < x \leq 8$.
5. $x \geq -\sqrt{3}$.
6. $x \leq \dfrac{3}{4}$.
7. $-\dfrac{2}{3} < x < 1$.
8. $x < \sqrt{2}$.
9. $-\sqrt{5} \leq x$.
10. $-1 \leq x \leq 5$.
11. $x \leq 23$.
12. $0 \leq x$.

Escribir los siguiente intervalos como una desigualdad y representarlos geométricamente:

13. $[-9, +\infty)$.
14. $[-10, -1)$.
15. $\left(\dfrac{5}{7}, +\infty\right)$.
16. $(-2, 16]$.
17. $(-\infty, 32)$.
18. $\left(\dfrac{1}{3}, 15\right)$.
19. $\left(-\infty, \dfrac{15}{4}\right]$.
20. $\left[-\dfrac{4}{3}, \dfrac{9}{2}\right]$.

Expresar como una desigualdad y con notación de intervalo los siguientes segmentos de la recta numérica:

21.

22.

23.

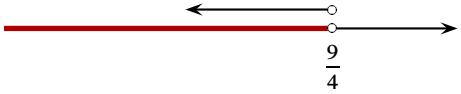
24.

25.

26.

27.

28.



Dados los intervalos $I_1 = (-7, 4]$, $I_2 = [-2, 6)$, $I_3 = (-\infty, 1]$, $I_4 = (0, +\infty)$, $I_5 = (-4, 2)$ e $I_6 = [2, 8]$, determinar:

29. $I_1 \cup I_2$.
30. $I_1 \cup I_6$.
31. $I_1 \cap I_2$.
32. $I_2 \cap I_6$.
33. $I_1 - I_2$.
34. $I_2 - I_5$.
35. $I_3 \cap I_4$.
36. $I_4 \cap I_5$.
37. $I_4 \cap I_6$.
38. $I_1 \cup I_5$.
39. $\mathbb{R} - I_3$.
40. $\mathbb{R} - I_4$.
41. $\mathbb{R} - I_2$.
42. $I_1 \cap I_6$.
43. $I_3 \cup I_4$.
44. $\mathbb{R} - I_1$.
45. $I_4 - I_6$.
46. $(I_5 \cap I_6) \cup I_4$.
47. $(I_1 \cap I_5) \cup I_6$.
48. $I_3 \cap (\mathbb{R} - I_5)$.

Ejercicios 1.5.1 *Intervalos, página 6*

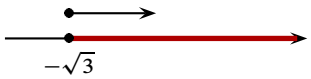
1. $[-4, 3)$.

2. $(-12, +\infty)$.

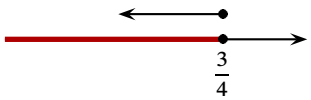
3. $(-\infty, 0)$.

4. $(\pi, 8]$.

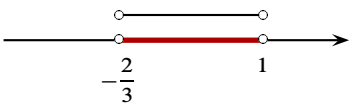
5. $\left[-\sqrt{3}, +\infty\right)$.



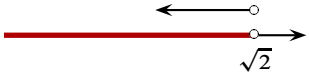
6. $\left(-\infty, \frac{3}{4}\right]$.



7. $\left(-\frac{2}{3}, 1\right)$.



8. $\left(-\infty, \sqrt{2}\right)$.

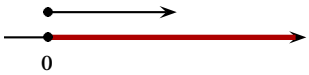


9. $\left[-\sqrt{5}, +\infty\right)$.

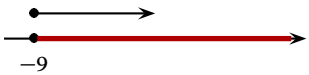
10. $[-1, 5]$.

11. $(-\infty, 23]$.

12. $[0, +\infty)$.



13. $\{\, x \mid -9 \leq x \,\}$.



14. $\{\, x \mid -10 \leq x < -1 \,\}$.

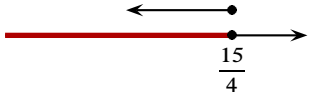
15. $\left\{\, x \mid \frac{5}{7} < x \,\right\}$.

16. $\{ x \mid -2 < x \leq 16 \}$.

17. $(-\infty, 32) = \{ x \mid x < 32 \}$.

18. $\left\{ x \mid \frac{1}{3} < x < 15 \right\}$.

19. $\left\{ x \mid x \leq \frac{15}{4} \right\}$.



20. $\left\{ x \mid -\frac{4}{3} \leq x \leq \frac{9}{2} \right\}$.

21. $(-13, +\infty) = \{ x \mid -13 < x \}$.

22. $(1, 22] = \{ x \mid 1 < x \leq 22 \}$.

23. $(-\infty, 6] = \{ x \mid x \leq 6 \}$.

24. $\left(-16, -\frac{3}{2}\right) = \left\{ x \mid -16 < x < -\frac{3}{2} \right\}$.

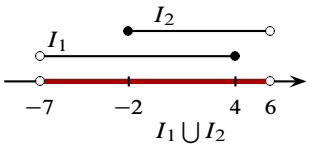
25. $\left[0, \frac{8}{3}\right] = \left\{ x \mid 0 \leq x \leq \frac{8}{3} \right\}$.

26. $[-1, +\infty) = \{ x \mid -1 \leq x \}$.

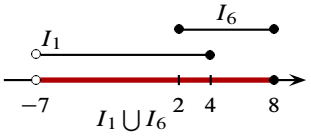
27. $[-5, 5) = \{ x \mid -5 \leq x < 5 \}$.

28. $\left(-\infty, \frac{9}{4}\right) = \left\{ x \mid x < \frac{9}{4} \right\}$.

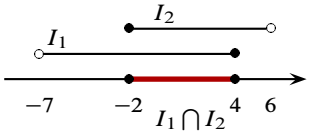
29. $(-7, 6)$.



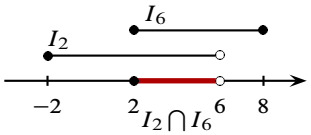
30. $(-7, 8]$.



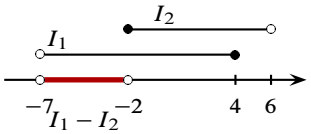
31. $[-2, 4]$.



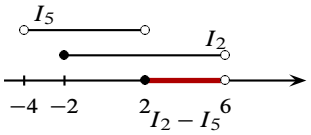
32. $[2, 6)$.



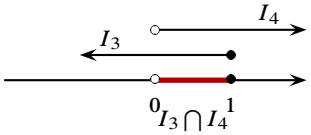
33. $(-7, -2)$.



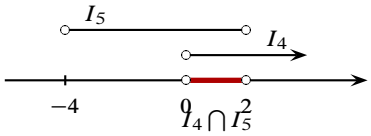
34. $[2, 6)$.



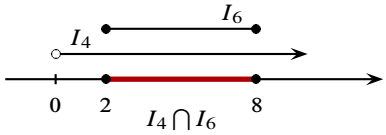
35. $(0, 1]$.



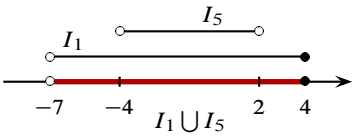
36. $(0, 2)$.



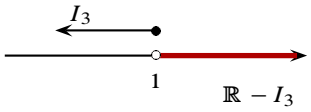
37. $[2, 8]$.



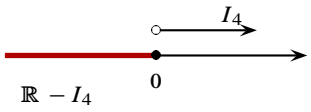
38. $(-7, 4]$.



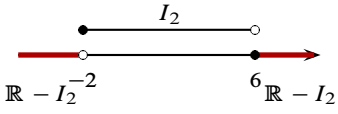
39. $(1, +\infty)$.



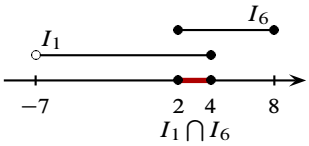
40. $(-\infty, 0)$.



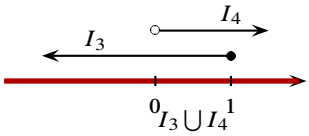
41. $(-\infty, -2) \cup [6, +\infty)$.



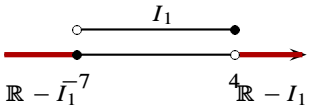
42. $[2, 4]$.



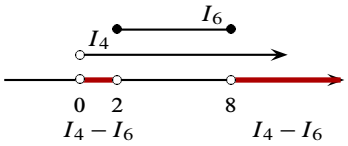
43. $\mathbb{R}$.



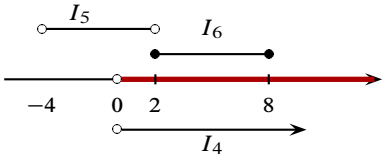
44. $(-\infty, -7] \cup (4, +\infty)$.



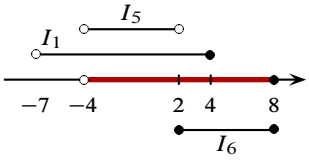
45. $(0, 2) \cup (8, +\infty)$.



46. $(0, +\infty)$.



47. $(-4, 8]$.



48. $(-\infty, -4]$.